Funkcia jednej reálnej premennej

1. Určte definičný obor funkcie

1. $f(x) = \dfrac{8}{x-3}$ $[\mathbb{R} \setminus \{3\}]$

2. $f(x) = \dfrac{11+7x}{5} + \dfrac{1}{13-x}$ $[\mathbb{R} \setminus \{13\}]$

3. $f(x) = \dfrac{x-4}{13x^2+10x-3}$ $\left[\mathbb{R} \setminus \left\{-1; \frac{3}{13}\right\}\right]$

4. $f(x) = \dfrac{1}{|x+5|-1}$ $[\mathbb{R} \setminus \{-6; -4\}]$

5. $f(x) = \sqrt{2x-121} - \sqrt{2x+98}$ $\left[\left\langle \frac{121}{2}; \infty \right)\right]$

6. $f(x) = \dfrac{\sqrt{x+1}-3}{5-6x}$ $\left[\left\langle -1; \frac{5}{6}\right) \cup \left(\frac{5}{6}; \infty\right)\right]$

7. $f(x) = \sqrt{\dfrac{11-3x}{\sqrt{4x^2-19x+12}}}$ $\left[\left(-\infty; \frac{3}{4}\right)\right]$

8. $f(x) = \dfrac{\sqrt{1+x}-\sqrt{2-x}}{5^{-2x}}$ $[\langle -1; 2\rangle]$

9. $f(x) = \log \frac{x+3}{x-2}$ $[(-\infty; -3) \cup (2; \infty)]$

10. $f(x) = \ln \frac{1}{x^2-7x+10}$ $[(-\infty; 2) \cup (5; \infty)]$

11. $f(x) = \log_2 \frac{3-x}{2x^2-11x+5}$ $\left[\left(-\infty; \frac{1}{2}\right) \cup (3; 5)\right]$

12. $f(x) = 5^{\log \frac{1}{3x-x^2}}$ $[(0; 3)]$

13. $f(x) = \sqrt{\dfrac{\log_5 x}{3^x-1}}$ $[(1; \infty)]$

14. $f(x) = \dfrac{1-\cos 2x}{2\sin x}$ [všetky reálne čísla $x \neq k\pi$, kde $k \in \mathbb{Z}$]

15. $f(x) = \dfrac{\sin 3x}{\sin 4x}$ [všetky reálne čísla $x \neq k\dfrac{\pi}{4}$, kde $k \in \mathbb{Z}$]

16. $f(x) = \dfrac{1-\operatorname{tg} x}{x}$ $\left[\mathbb{R} \setminus \{0; (2k+1)\frac{\pi}{2}, \text{ kde } k \in \mathbb{Z}\}\right]$

17. $f(x) = 7^{\frac{5-x}{2x-7}}$ $\left[\mathbb{R} \setminus \left\{\frac{7}{2}\right\}\right]$

18. $f(x) = \sqrt{x^2-7x+10}$ $[(-\infty; 2) \cup \langle 5; \infty)]$

19. $f(x) = \dfrac{2x+3}{\sqrt{x^2-x-2}}$ $[(-\infty; 1) \cup (2; \infty)]$

20. $f(x) = \log \frac{1-x}{1+x}$ $[(-1; 1)]$

21. $f(x) = \ln \frac{7-x}{7+x}$ $[(-7; 7)]$

22. $f(x) = \dfrac{7x+3}{\log(6-x)}$ $[(-\infty; 5) \cup (5; 6)]$

23. $f(x) = \sqrt{3}\frac{1}{x+2} + \log(x^2+7x+10)$ $[(-\infty; -5) \cup (-2; \infty)]$

24. $f(x) = \arcsin \frac{2+3x}{2}$ $\quad \left[\left\langle -\frac{4}{3}; 0 \right\rangle \right]$

25. $f(x) = \arccos \frac{x-3}{5}$ $\quad [\langle -2; 8 \rangle]$

26. $f(x) = \sqrt{\log(4-x)}$ $\quad [(-5; -4\rangle]$

27. $f(x) = \sqrt{\log_{\frac{1}{3}}(5+x)}$ $\quad [(-\infty; 3]]$

28. $f(x) = \sqrt{\frac{x-1}{x+2}} + \sqrt{\frac{x+1}{x-2}}$ $\quad [(-\infty; -2\rangle \cup \langle 2; \infty)]$

29. $f(x) = \sqrt{\sin x} + \sqrt{9-x^2}$ $\quad [\langle 0; 3 \rangle]$

30. $f(x) = \operatorname{arctg} \frac{x-1}{1+x}$ $\quad [(-\infty; -1) \cup (1; \infty)]$

2. Zistite, či sa funkcie f,g rovnajú

1. $f(x) = \sqrt{2x(x-2)}$, $g(x) = \sqrt{x}\sqrt{2(x-2)}$, $\quad$ [no]

2. $f(x) = 5$, $g(x) = \dfrac{5x}{x}$ $\quad$ [áno]

3. $f(x) = |x-1|$, $g(x) = |1-x|$ $\quad$ [áno]

4. $f(x) = x$, $g(x) = \dfrac{\sqrt{4x^2}}{2}$ $\quad$ [nie]

3. Vyšetrite monotónnosť funkcie f

1. $f(x) = 2x + 1$ $\quad$ [rastúca na $D(f)$]

2. $f(x) = 3^x + 2$ $\quad$ [rastúca na $D(f)$]

3. $f(x) = -\log_3 x + 4$ $\quad$ [klesajúca na $D(f)$]

4. $f(x) = x^2 + 4x + 3$ $\quad$ [klesá na $(-\infty; -2\rangle$ a rastie na $\langle -2; \infty)$]

5. $f(x) = 1 - \dfrac{3}{x}$ $\quad$ [rastúca na $D(f)$]

6. $f(x) = \dfrac{x+2}{7-x}$ $\quad$ [rastúca na $D(f)$]

7. $f(x) = -\dfrac{x}{4} + 3$ $\quad$ [klesajúca na $D(f)$]

8. $f(x) = |x+1|$ $\quad$ [klesá na $(-\infty; -1\rangle$ a rastie na $\langle -1; \infty)$]

4. Zistite, ktorá z uvedených funkcií je párna alebo nepárna

1. $f(x) = x^2 + 7$ $\quad$ [párna]

2. $f(x) = \dfrac{x^2 + 3x}{x+3}$ $\quad$ [ani párna ani nepárna]

3. $f(x) = 3x^2 - x^4$ $\quad$ [párna]

4. $f(x) = 14x^3 + 13x^2 - 12x + 11$ $\quad$ [ani párna ani nepárna]

5. $f(x) = \sqrt{3}x + 1$ [ani párna ani nepárna]

6. $f(x) = x^4 + \dfrac{1}{\sqrt{4(2x)^2}}$ [párna]

7. $f(x) = \dfrac{\sin x}{2x}$ [párna]

8. $f(x) = \dfrac{\cos x}{x}$ [nepárna]

5. Vyšetrite ohraničenosť funkcie f

1. $f(x) = x^3 + 1$ [neohraničená]

2. $f(x) = \dfrac{2x + 1}{x + 3}$ [neohraničená]

3. $f(x) = \dfrac{2x}{1 + x^2}$ [ohraničená]

4. $f(x) = \dfrac{x - 1}{(2x - 3)(x - 2)}$, $x \in (3; \infty)$ [ohraničená]

6. Zistite, či existuje inverzná funkcia k funkcii f a ak áno, nájdite ju

1. $f(x) = 2x - 3$ $\left[f^{-1}(x) = \dfrac{x + 3}{2} \right]$

2. $f(x) = \dfrac{2x - 1}{x + 8}$ $\left[f^{-1}(x) = \dfrac{17}{2 - x} + 8 \right]$

3. $f(x) = \sqrt{4x^2 - 1}$ [neexistuje]

4. $f(x) = 5^{x+2} - 4$ $\left[f^{-1}(x) = \log_5 \dfrac{x + 4}{25} \right]$

5. $f(x) = \ln(2 - 3x)$ $\left[f^{-1}(x) = \dfrac{2 - e^x}{3} \right]$

6. $f(x) = 3^{1 + \mathrm{arctg}\ x}$ $\left[f^{-1}(x) = \mathrm{tg}(\log_3 x - 1) \right]$

7. $f(x) = \dfrac{2x - 1}{5x + 2}$ $\left[f^{-1}(x) = \dfrac{1 + 2x}{2 - 5x} \right]$

8. $f(x) = \log(3 - 2x)$ $\left[f^{-1}(x) = \dfrac{3 - 10^x}{2} \right]$

9. $f(x) = 7 + \sqrt{2 + e^{4x}}$ [neexistuje]

10. $f(x) = 12^{\frac{x}{x-1}}$ $\left[f^{-1}(x) = \dfrac{1}{\log_{12} x - 1} + 1 \right]$

11. $f(x) = 5 + 2\cos \frac{x-1}{x+2}$ $\left[f^{-1}(x) = \dfrac{3}{1 - \arccos \frac{x-5}{2}} - 2 \right]$

12. $f(x) = 5 + 4\arcsin \frac{4x-1}{5}$ $\left[f^{-1}(x) = \frac{5}{4}\sin \frac{x-5}{4} + \frac{1}{4} \right]$

13. $f(x) = 5 + 3\,\mathrm{arctg}\, \frac{2x+3}{12}$ $\left[f^{-1}(x) = 6\,\mathrm{tg}\, \frac{x-5}{3} - \frac{3}{2} \right]$

14. $f(x) = \dfrac{2.5^x - 5}{3.5^x + 2}$ $\left[f^{-1}(x) = \log_5 \frac{5+2x}{2-3x} \right]$

15. $f(x) = 5 + 4^{\frac{x-1}{x}}$ $\left[f^{-1}(x) = \dfrac{1}{1 - \log_4(x-5)} \right]$

16. $f(x) = \log_3 \frac{x+6}{x-1}$ $\left[f^{-1}(x) = \dfrac{3^x + 6}{3^x - 1} \right]$

7. Načrtnite graf funkcie f

1. $f(x) = x + 5$

2. $f(x) = -x + 8$

3. $f(x) = (x+3)^2 + 3$

4. $f(x) = x^2 + 4x + 5$

5. $f(x) = -x^2 + 2x + 8$

6. $f(x) = \dfrac{2x + 5}{x + 2}$

7. $f(x) = 1 + \log(x + 3)$

8. $f(x) = \ln(x + 2)$

9. $f(x) = \left(\dfrac{22}{33} \right)^x - 2$

10. $f(x) = e^x - 11$